The Exponential Transmission Line *

By CHAS. R. BURROWS

The theory of the exponential transmission line is developed. It is found to be a high pass, impedance transforming filter. The cutoff frequency depends upon the rate of taper.

The deviation of the exponential line from an ideal impedance transformer may be decreased by an order of magnitude by shunting the low impedance end with an inductance and inserting a capacitance in series with the high impedance end. The magnitudes of these reactances are equal to the impedance level at their respective ends of the line at the cutoff frequency.

For a two-to-one impedance transformer the line is 0.0551 wave-lengths long at the cutoff frequency. For a four-to-one impedance transformer the line is 0.1102 wave-lengths long at the cutoff frequency, etc.

The results have been verified experimentally. Practical lines 50 meters and 15 meters long have been constructed which transform from 600 to 300 ohms over the frequency range from 4 to 30 mc. with deviations from the ideal that are small compared with the deviations from the ideal of commercial transmission lines, either two-wire or concentric.

When an exponential line is used as a dissipative load of known impedance instead of a uniform line it is possible to approach more nearly the ideal of constant heat dissipation per unit length. This makes it possible to use a shorter line.

THE exponential line may be defined as an ordinary transmission line in which the spacing between the conductors (or conductor size) is not constant but varies in such a way that the distributed inductance and capacitance vary exponentially with the distance along the line. That is, the impedance ratio for two points a fixed distance apart is independent of the position of these two points along the line. A disturbance is propagated down an exponential transmission line in the same manner as it would be down a uniform line with the additional effect that the voltage is increased by the square root of the change in impedance level and the current is decreased by the reciprocal of this quantity.

The exponential line has the properties of a high pass impedance transforming filter. The cutoff frequency depends upon the rate of

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taper. As the frequency is increased the transfer constant * approaches the propagation constant of the equivalent uniform line. At sufficiently low frequencies the only effect of the line is to connect the input to the load.

Above cutoff the magnitudes of the characteristic impedances at any point are approximately equal to the nominal characteristic impedance * at that point but their phase angles (in radians) differ by an amount which at the higher frequencies is equal to the cutoff frequency divided by the frequency in question. The ratio of input impedance to the input impedance level * of an exponential line terminated in a resistance equal to the impedance level at the output always remains within the range from $1 - f_1/f$ to $1/(1 - f_1/f)$ for frequencies, f, greater than the cutoff frequency, f_1. For a 2 : 1 transformation this means that the input impedance remains within $\pm$ 6 per cent of the desired value for all frequencies above that for which the line is a wave-length long. For a 4 : 1 transformation under the same conditions the irregularities are twice as great.

A transforming network having deviations from the ideal of the order of $\pm (f_1/f)^2$ may be made by connecting an inductance in parallel with the low impedance terminal and a capacitance in series with the high impedance terminal. The magnitudes of these reactances are such that their impedances are equal to the impedance levels of the line at their respective ends at the cutoff frequency. Or expressed in another way the capacitance is equal to $2/(k - 1)$ times the electrostatic capacitance of the line and the inductance is the same factor times the total loop inductance of the line where k is the impedance transformation ratio of the line.

Figure 1 shows the theoretical input impedance-frequency characteristics for 2 to 1 step-up and step-down exponential lines. Curve 1 is for the line with a resistance termination. At low frequencies the input impedance is equal to the load impedance while at high frequencies the line approaches an ideal transformer. Curve 2 is the input impedance of the line terminated with the appropriate resistance-reactance combination. The improvement in the input impedance characteristic for frequencies above the cutoff frequency is evident. At the lower frequencies the input impedance does not approach the terminal reactance but approaches the reactance of the capacitance of the line in parallel with the series terminal capacitance for the step-up line and the reactance of the inductance of the line in series with the shunt terminal inductance for the step-down line. The improvement is not as great as apparent from the figures because the phase angle is

* See appendix for definition of terms.

not improved proportionally. This is easily remedied by completing the impedance transforming network with the appropriate reactance at the input. The resulting input impedance is shown in curve 3. In the "pass" frequency range the maximum reactive component is of

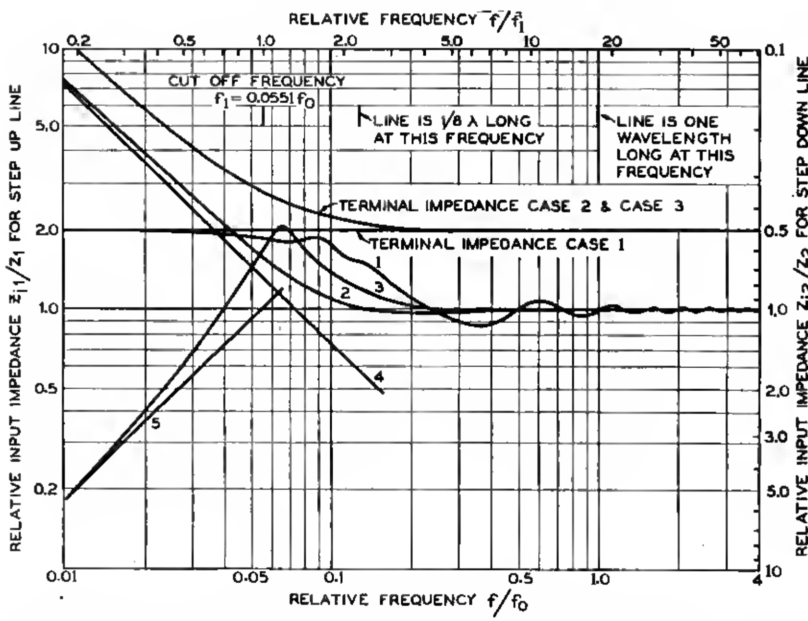

Fig. 1—Input impedance characteristics of 1 : 2 exponential lines. Left ordinate scale refers to step-up line. Right ordinate scale refers to step-down line.

Curve 1—Resistance termination.

Curve 2—With capacity equal to twice the electrostatic capacity of the line in series with the same resistance, $Z_2 = Z_2(1 - jf_1/f)$, for step-up line, or with an inductance equal to twice the total inductance of the line in shunt with the same resistance, $Z_1 = Z_1/(1 - jf_1/f)$, for step-down line.

Curve 3—Termination as for curve 2 with inductance equal to twice the total inductance of the line in parallel with input to the line, $Z_{i1} = Z_1/(1 - jf_1/f)$, for step-up line, or termination as for curve 2 with capacity equal to twice the static capacity of line in series with input to the line, $Z_{i2} = Z_2(1 - jf_1/f)$.

Curve 4—Asymptotic value of impedance of capacity of line in parallel with termination for case 2 for step-up line, or asymptotic value of impedance of inductance in series with termination for case 2 for step-down line.

Curve 5—Impedance of shunt inductance added at input for case 3 for step-up line, or impedance of capacity added in series at input for case 3 for step-down line.

the same order of magnitude as the deviation of the impedance from the ideal.

Besides its application as an impedance transforming network, the exponential line may be used as a "resistance" load of constant known impedance that has a high capability for dissipating power. As such it is capable of dissipating more power in the same length of line than

the uniform line. If x is the maximum attenuation in nepers that can be obtained with a uniform line without overheating, the same length of exponential line will have an attenuation of $(e^{2x} - 1)/2$ nepers.

Exponential lines of the proper length have properties similar to half-wave and quarter-wave uniform lines. The input impedance of an exponential line an even number of quarter wave-lengths long is equal to the load impedance times the impedance transformation ratio of the line. When the length of the line differs from an odd multiple of a quarter wave-length by an amount that depends upon the frequency and load impedance, the input impedance is equal to the product of the terminal impedance levels divided by the load impedance.

Mathematical Formulation

The telegraph equations for the exponential line may be solved by the methods employed in the problem of a uniform line. The resulting equations for the voltage and current at any point along the line are

$$v_x = Ae^{-\left(\Gamma - \frac{\delta}{2}\right)x} + Be^{+\left(\Gamma + \frac{\delta}{2}\right)x} = Ae^{-\left(\Gamma - \frac{\delta}{2}\right)x}\left[1 + \frac{B}{A}e^{2\Gamma x}\right] \tag{1}$$

and

$$i_x = \frac{A}{Z_0}\frac{\Gamma - \frac{\delta}{2}}{\gamma}e^{-\left(\Gamma + \frac{\delta}{2}\right)x} - \frac{B}{Z_0}\frac{\Gamma + \frac{\delta}{2}}{\gamma}e^{+\left(\Gamma - \frac{\delta}{2}\right)x}$$

$$= \frac{A}{Z_0}\frac{\Gamma - \frac{\delta}{2}}{\gamma}e^{-\left(\Gamma + \frac{\delta}{2}\right)x}\left[1 - \frac{B}{A}\frac{\Gamma + \frac{\delta}{2}}{\Gamma - \frac{\delta}{2}}e^{2\Gamma x}\right], \tag{2}$$

where

$\delta = \dfrac{\log_e z/z_0}{x} = \dfrac{\log_e y_0/y}{x} = \dfrac{\log_e Z/Z_0}{x}$ is the rate of taper,

$Z_x = \sqrt{z/y} = Z_0 e^{\delta x}$ is the *surge* or *nominal characteristic impedance* of the exponential line at the point x which is equal to the characteristic impedance of the uniform line that has the same distributed constants as this line has at the point x,

$\gamma = \sqrt{zy} = \sqrt{z_0 y_0}$ is the *propagation constant* of any uniform line that has the same distributed constants as this line at any point. It is independent of the point along the line to which it is referred, and

$\Gamma = \sqrt{\gamma^2 + \delta^2/4} = \alpha + j\beta$ is the *transfer constant* of the exponential line.

$+\gamma$ and $+\Gamma$ refer to the values of the indicated roots that are in the first quadrant.

If these equations are compared with those for a uniform transmission line it is found that the *propagation constant* is $\Gamma - \delta/2$ for voltage waves traveling in the positive x direction and $\Gamma + \delta/2$ for voltage waves traveling in the negative x direction. For current waves the corresponding *propagation constants* are $\Gamma + \delta/2$ and $\Gamma - \delta/2$. In the terminology of wave filters, Γ is the *transfer constant* and δ is the impedance *transformation constant*. $\delta/2$ is the *voltage transformation constant* and $-\delta/2$ is the *current transformation constant*. The real and imaginary parts of Γ, α and β are the *attenuation* and *phase constants* respectively.

An important parameter is

$$\nu = j\frac{\delta}{2\gamma},$$

which for a non-dissipative line is the ratio of the cutoff frequency to the frequency, as can be seen if we write the transfer constant as

$$\Gamma = \gamma\sqrt{1 - \nu^2},$$

where the indicated root is in the fourth quadrant. For a non-dissipative line ν is real and the transfer constant is real or imaginary depending on whether ν^2 is greater than or less than unity. Hence the exponential line is a high pass filter whose cutoff frequency, f_1, is that frequency for which $\nu = \pm 1$. The transfer constant is then less than that for a uniform line by the factor $\sqrt{1 - \nu^2}$ so that both phase velocity and wave-length are larger for the exponential line than for the uniform line by the reciprocal of this factor.

If we terminate this line at $x = l$ with an impedance $Z_l = v_l/i_l$, the ratio of the reflected to direct voltage wave is found to be

$$\frac{B}{A} = -\frac{1 - (Z_l/Z_l)(\sqrt{1 - \nu^2} + j\nu)}{1 + (Z_l/Z_l)(\sqrt{1 - \nu^2} - j\nu)}e^{-2\Gamma l}, \tag{3}$$

where the coefficient of the exponential is the *voltage reflection coefficient*.

There will be no reflection if

$$Z_l = Z_l/(\sqrt{1 - \nu^2} + j\nu) = Z_l^{+}, \tag{4}$$

which becomes $Z_l e^{-j\sin^{-1}\nu}$ above the cutoff frequency for non-dissipative lines. This is the magnitude of the forward-looking *characteristic impedance* at $x = l$ as can be seen by dividing the first term of (1) by the first term of (2). Curve 1 of Fig. 2 gives the charac-

teristic impedance of a non-dissipative exponential line looking toward the high impedance end as a function of frequency. At infinite frequency the characteristic impedance is a resistance equal to the nominal characteristic impedance but as the frequency is decreased the phase angle of the characteristic impedance changes so that its locus

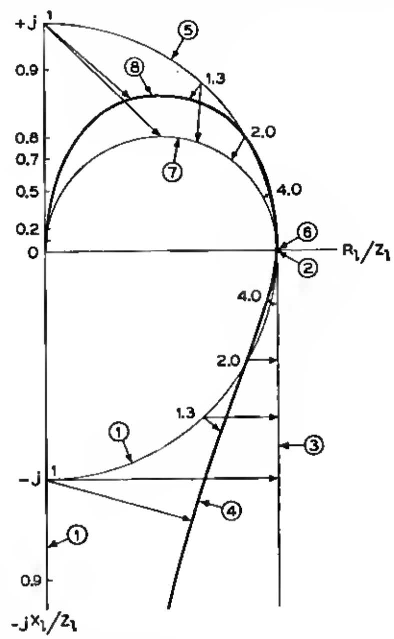

Fig. 2—Impedance diagram comparing the forward looking characteristic impedance with various terminal impedances. The numbers give the frequency relative to cutoff. The arrows are the vectors $Z_t - Z_l^+$ which are a measure of the magnitude of the reflection.

A. Step-up line.

Curve 1—Forward looking characteristic impedance,

$$Z_l^+ = Z_l e^{-j\sin^{-1}(f_1/f)}, \qquad f > f_1,$$
$$Z_l^+ = Z_l[-j(f_1/f)(1 + \sqrt{1 - f^2/f_1^2})], \qquad f_1 > f;$$

Curve 2—Resistance termination, $Z_t = Z_l$;
Curve 3—Capacity resistance termination, $Z_t = Z_l(1 - jf_1/f)$;
Curve 4—Capacity, resistance and inductance termination adjusted for no reflection at twice the cutoff frequency and at infinite frequency;

B. Step-down line.

Curve 5—Forward-looking characteristic impedance,

$$Z_l^+ = Z_l e^{+j\sin^{-1}(f_1/f)}, \qquad f > f_1,$$
$$Z_l^+ = Z_l[+j(f_1/f)(1 - \sqrt{1 - f^2/f_1^2})], \qquad f_1 > f;$$

Curve 6—Resistance termination $Z_t = Z_l$;
Curve 7—Inductance resistance termination $Z_t = Z_l/(1 - jf_1/f)$;
Curve 8—Inductance, resistance and capacity termination adjusted for no reflection at twice the cutoff frequency and at infinite frequency.

is the circular arc. At and below cutoff it is a pure reactance. If the load is a resistance equal to the nominal characteristic impedance at the terminal as indicated at 2 of Fig. 2, there will be no reflection at infinite frequency, but as the frequency is lowered there will be an increasing impedance mismatch with its accompanying reflected wave.

This reflection may be materially reduced by inserting a condenser in series with the resistance load as shown by curve 3. Further improvement results from more complicated networks. Curve 4 shows the effect of adding an inductance in shunt with the resistance load of the resistance-capacitance combination. The arrows indicate the resulting impedance mismatch which is a measure of the reflected wave.

The characteristic impedance looking toward the low impedance end is the inverse of that looking in the other direction as shown by curve 5. Shunting the resistance load with an inductance gives the impedance curve 7. Adding a capacitance element gives curve 8.

Division of (1) by (2) and substitution of the result of (3) gives the following ratio for the impedance looking into the line at the point x to the impedance level at that point,

$$\frac{Z_x}{Z_x} = \frac{K(\sqrt{1-\nu^2} - j\nu) + 1 + [K(\sqrt{1-\nu^2} + j\nu) - 1]e^{-2\Gamma(l-x)}}{K + j\nu + \sqrt{1-\nu^2} - [K - \sqrt{1-\nu^2} + j\nu]e^{-2\Gamma(l-x)}}, \qquad (5)$$

where $K = Z_l/Z_l$ is the ratio of the load impedance to the impedance level at the terminal. Here as before the indicated root is in the fourth quadrant.

Network Characteristics

Three parameters are required to specify the characteristics of an exponential line of negligible loss: (1) the cutoff frequency, f_1, (2) the length of the line which is perhaps best specified as the frequency, f_0 = velocity of light/length of line, for which the line is one wavelength long, and (3) the impedance level at some point along the line. We will designate the impedance levels at the low and high impedance ends of the line by Z_1 and Z_2 respectively, and their ratio Z_2/Z_1 by k.

When the line is terminated in a resistance equal to the impedance level at the output (5) reduces to

$$\frac{Z_1}{Z_1} = k^{\cos 2\zeta} e^{2j\zeta} \frac{1 + j \tan \zeta \, k^{-\cos 2\zeta}}{1 - j \tan \zeta \, k^{\cos 2\zeta}}, \qquad \nu > 1, \qquad (6)$$

$$\frac{Z_1}{Z_1} = e^{-2j\xi} \frac{1 + \tan \xi \, e^{j\left(2\xi + \frac{\pi}{2} - \eta\right)}}{1 + \tan \xi \, e^{j\left(-2\xi - \frac{\pi}{2} - \eta\right)}}, \qquad \nu < 1. \qquad (7)$$

for frequencies below and above cutoff respectively. Here $\eta = -j2\Gamma l$ is twice the electrical length of the line in radians, $\sin 2\zeta = 1/\nu$, $\sin 2\xi = \nu$ and $\cos 2\xi$ is ratio of the electrical length of the line to that of a uniform line of the same physical length. For the step-down line the corresponding ratios are the reciprocal of the above expressions. These ratios are plotted in Fig. 1.

When $f \to 0$, $Z_1 = kZ_1 = Z_2 = Z_2$ and the only effect of the line is to connect the load to the input. Above cutoff the magnitude of the input impedance oscillates about the nominal characteristic impedance and the phase angle oscillates about the value $-2\xi(\approx -f_1/f$ for $f \gg f_1)$ which goes from $-\pi/2$ to 0 as the frequency increases indefi-

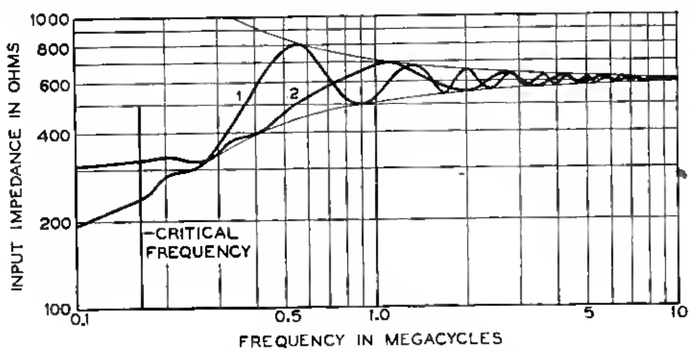

Fig. 3—Input impedance characteristics.
Curve 1—150 : 600 ohm line, 100 meters long.
Curve 2—300 : 600 ohm line, 200 meters long.
Both lines have the same rate of taper.

nitely from cutoff. The variation of the input impedance with frequency is shown for two lines of different length but the same rate of taper in Fig. 3. The magnitude of the oscillations depends only on the rate of taper and decreases with increase in frequency. The impedance varies between $(1 + f_1/f)$ and $1/(1 + f_1/f)$. The positions of the maxima and minima, however, are determined by the length of the line. They occur respectively at those frequencies for which the line is approximately 1/8 of a wave-length more than an even or an odd number of quarter wave-lengths long. The phase angle is usually negative but has a small positive value when the line is approximately a half wave-length long.

The locations of these maxima and minima are the same as would result from terminating a uniform line in an impedance whose magnitude is the same as the characteristic impedance but has a small reactive component. This suggests adding a compensating reactance

to the resistance load. From (3) the best single reactive element is found to be a condenser whose impedance is equal to the impedance level at the cutoff frequency. This gives a value of $K = 1 - j\nu$ which when substituted in (5) shows that the input impedance is to a first approximation a constant times the terminal impedance. To correct for the reactive component of the input impedance an inductance having an impedance jZ_1/ν which is equal to the input impedance level at cutoff is shunted across the input. The resulting impedance transforming network consists of an exponential line with a series capacitance at the high impedance end and a shunt inductance at the low impedance end. When terminated in a resistance load at either end equal to the impedance level at that end the input impedance, to a first approximation, is a resistance equal to the impedance level at the input end. In fact the deviations of the input impedance from the ideal for transmission in one direction are just the reciprocal of those for transmission in the other direction.

The magnitudes of the series capacitance and shunt inductance that give the improved network may be expressed in terms of the electrostatic capacitance and loop inductance of the line. Simple calculation shows that the required series capacitance is equal to $2/(k - 1)$ times the electrostatic capacitance of the line and the required shunt inductance is equal to the same factor times the total inductance of the line.

There is an interesting relationship between these terminations and a simple high-pass filter. The LC product of the shunt and series arms of the filter resonates at f_1. If an ideal transformer with transformation ratio k is inserted between the shunt inductance and the series capacitance, the capacitance becomes C/k and the new LC resonates at $f_1\sqrt{k}$. This is the same frequency at which the series capacitance and shunt inductance that are added to the terminations of the exponential line resonate. Furthermore the reactance of the shunt inductance is equal to the impedance level at the cutoff frequency and the reactance of the series capacitance is equal to the impedance level at the cutoff frequency exactly as in the case of the high-pass filter.

By using the exponential line it is possible to construct a network with properties that no network with lumped circuit elements possesses, namely, a high-pass impedance transforming filter.

Critical Lengths

Besides the characteristics of the exponential line that are substantially independent of the length of the line, it has properties that

depend on the length of the line that are analogous to those of a uniform line a half wave-length or quarter wave-length long. For non-dissipative lines above the cutoff frequency (5) becomes

$$Z_1 = \frac{K\cos\left(\frac{\eta}{2} - 2\xi\right) + j\sin\frac{\eta}{2}}{\cos\left(\frac{\eta}{2} + 2\xi\right) + jK\sin\frac{\eta}{2}} Z_1. \tag{8}$$

When the line is an integral number of half wave-lengths long ($2\eta = \pi$) this reduces to

$$Z_1 = KZ_1 = kZ_2, \tag{9}$$

which says that the input impedance is equal to the impedance transformation ratio times the load impedance. The length of exponential line that corresponds to a quarter-wave uniform line differs from an odd multiple of a quarter wave-length by an amount such that

$$\tan\left(\frac{\eta - (2n+1)\pi}{2}\right) = \frac{K^2 - 1}{K^2 + 1}\tan 2\xi, \tag{10}$$

for which (8) becomes

$$Z_1 = \frac{Z_1 Z_2}{Z_2}. \tag{11}$$

Similar expressions exist for the step-down line, but $1/K$ must be substituted for K in (10) for the length corresponding to the quarter-wave uniform line.

With Dissipation

An exponential line is an improvement over the uniform "iron wire" line as a resistance load that will dissipate a large amount of power.

Provided the attenuation is not too large the current and voltage distribution will be the same as for a non-dissipative line except for the additional power loss so that we may use the equations for an exponential line even though the distributed series resistance and shunt leakage do not vary exponentially with distance.

Suppose that the conductor size and resistance that will just dissipate the desired input power result in an attenuation constant α_0 for a uniform transmission line. To a first approximation the conductors can carry the same current irrespective of the impedance level. The current wave will be given by the first term of equation (2) which becomes

$$i = i_0 e^{-(\delta/2)x - \alpha_0 x},$$

except for a phase factor. In order that the current will not increase, $\delta = -2\alpha$. The actual attenuation "constant,"

$$\alpha_x \sim \left(\frac{R}{2Z_x} + \frac{GZ_x}{2}\right)\left(1 + \frac{1}{2}\frac{f_1^2}{f^2}\right) \sim \alpha_0 e^{2\alpha_0 x}\left(1 + \frac{1}{2}\frac{f_1^2}{f^2}\right), \quad (13)$$

will increase with distance down the line so that the current will decrease but not as rapidly as with a uniform line. The total attenuation in nepers is approximately

$$\left(1 + \frac{1}{2}\frac{f^2}{f_1^2}\right)\int_{x=0}^{l} \alpha_0 e^{2\alpha_0 x} dx = \left(1 + \frac{1}{2}\frac{f^2}{f_1^2}\right)\left(\frac{e^{2\alpha_0 l} - 1}{2}\right). \quad (14)$$

At the point where the attenuation of the uniform line is 6 db the tapered line has an additional attenuation of 7 db above the uniform line or a total attenuation of more than twice. The current has been reduced to less than half. Here an improvement may be made by increasing the dissipation by either changing the wire size or resistivity of the conductor. A greater improvement would result from changing the resistivity because then the capacity for heat dissipation would be the same. Suppose, however, that one conductor material is to be used throughout and the dissipation capacity is proportional to the wire surface; then at this point the wire size could be reduced to 1/2, doubling the attenuation factor. It is already 4 times that for the uniform line, so this increases it to 8 times. The resulting total attenuation is 30 db in a length that would have less than 7 db if the line were uniform. If this attenuation were required the length of line could be reduced by a factor of about 4.4. Of course the spacing is very close at the end of this line, but the line could be shorted at the end. This would approximately double the current at the end, but here again the current carrying capacity of the line is more than double the current traveling down the line. With the line shorted the reflected current would be 60 db down, which would not affect the input impedance appreciably. For the first 13 db of attenuation the impedance of the line would be relatively free from changes due to changes in spacing resulting from wind, etc. When the spacing is small enough to be affected by wind, vibration, etc., the attenuation will be great enough to suppress these small irregularities.

Experiment

In order to verify the foregoing theoretical development, measurements have been made on several experimental lines. Figure 4 shows the results of measurements on two such lines. These lines were

constructed of No. 12 tinned copper. At the low impedance end the strain was taken by a victron insulator which also served as a line spreader and terminal mounting. At the high impedance end the strain was taken by 1/4" manila rope without other insulation. The line spacing was adjusted by "lock stitch" tension insulators spaced 1 meter and 1/2 meter apart on the low and high impedance end respectively of the 9-meter line. The 3-meter line was supported at the 1/4, 1/2, 2/3, 3/4 and 7/8th points.

The impedance was measured by the substitution method. To facilitate the substitution of the reactive component of the line it was bridged by an antiresonant circuit. Pencil leads calibrated on direct current were used as the resistance standards. Type BW IRC 1/2

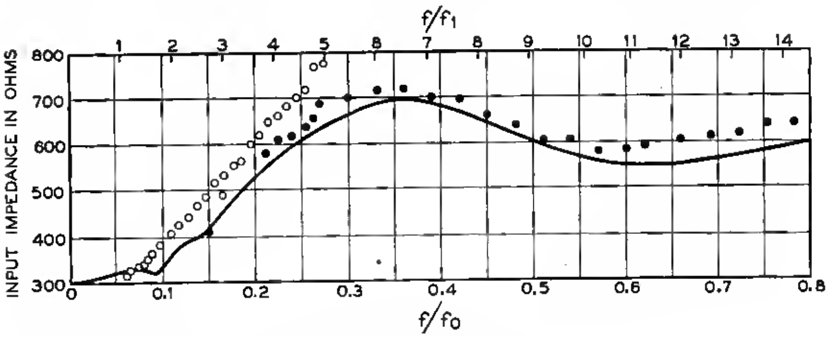

Fig. 4—Input impedance characteristic. Comparison of theoretical curve with experimental points for 600 : 300 ohm lines.
Solid circles—9-meter line.
Open circles—3-meter line.

watt resistances were used for terminations. The solid circles of Fig. 4 represent measurements on the 9-meter line. The agreement with theory is as good as is usually found for actual "uniform lines." In order to check the theory further toward the lower frequency end—beyond the range of the measuring equipment—measurements were made on a 3-meter line. These measurements are shown by the open circles. The agreement with theory is not so good, but here the lengths of the connecting leads are an appreciable fraction of the length of the line.

Preliminary tests on a full size model of exponential line impedance transformer showed deviations from the theoretical that might be attributed to improper termination, irregularities along the line, irregularities introduced at the change in conductor size or capacitance of the spacing insulators. Since it was impossible to determine which of these was the predominant cause of the deviations from the ideal, it

was decided to introduce each of these factors one at a time. This test was made on a 600 : 300 ohm line constructed of No. 6 copper wire with lockstitch insulators except at the terminals. The correct termination was obtained by tests on a uniform 300 ohm line with the same physical structure at the termination. Of necessity the tying of the wire to the strain insulators at the end introduced a shunt capacitance which augmented the inherent additional capacitance due the "end effect." This additional capacitance is equal to that of a short length of line.

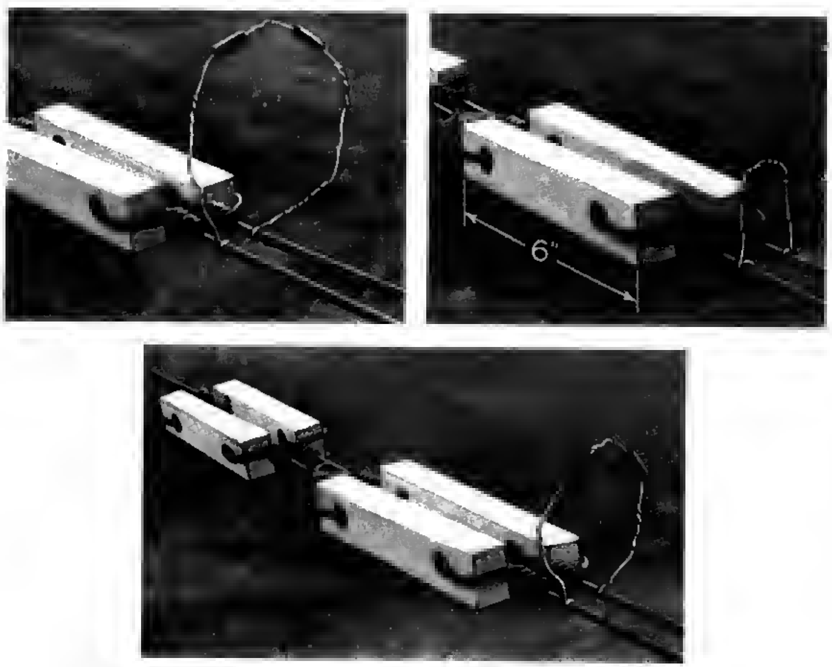

Fig. 5—Photographs of terminations of 300 ohm line.
upper right for curve 1 ⎫
lower for curve 2 ⎬ of Fig. 6.
upper left for curve 3 ⎭

If the correct amount of inductance is inserted in series with the resistance load the combined effect of the additional capacitance and inductance becomes the same as the addition of a small length of line for all frequencies up to those for which this length of line is an appreciable fraction of a wave-length. Accordingly a small amount of inductance was inserted in series with the resistance as shown in the right picture of Fig. 5. The input impedance of the uniform line with this termination is given by "Experimental Curve 1" at the bottom of Fig. 6.

A three-inch length of No. 18 wire was inserted as shown in the lower picture of Fig. 5 and "Experimental Curve 2" resulted. This reduced the irregularities in the input impedance to about half, so another three

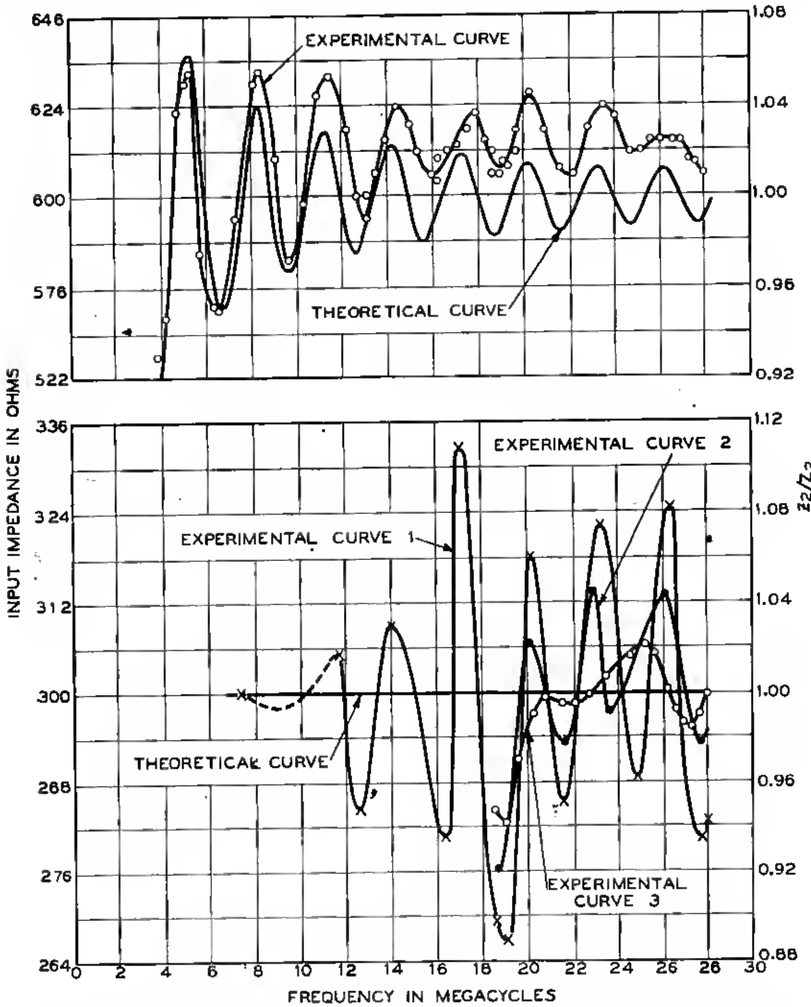

Fig. 6—Lower. Experimental input impedance characteristics of 300 ohm line with terminations shown in Fig. 5. Upper. Input impedance characteristics of 50-meter 600 : 300 ohm line of No. 6 conductors.

inches were inserted, resulting in "Experimental Curve 3." Here the maxima and minima are displaced, indicating that the effect of the stray capacitance has been reduced to the same order of magnitude as that due to the deviation of the resistance from the desired value. This

termination was accordingly removed to the exponential line, resulting in the "Experimental Curve" at the top of Fig. 6. It agrees within experimental error with the "Theoretical Curve." The slight vertical displacement of the experimental curve at the higher frequencies is attributed to deviations in the impedance of the pencil lead, which was

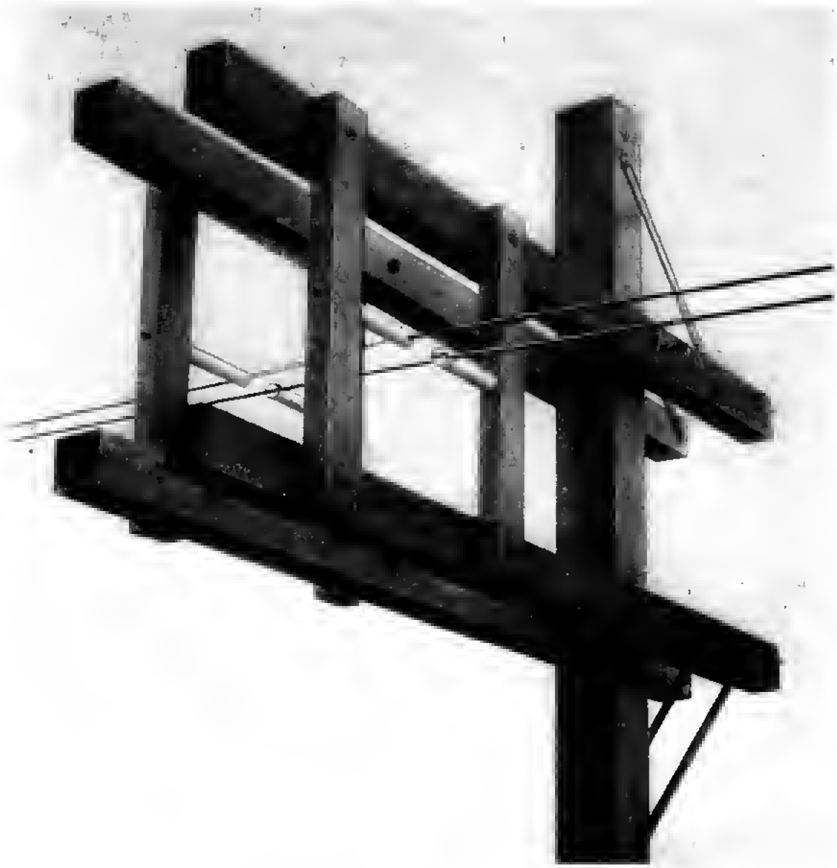

Fig. 7—Photograph of one of the changes in conductor size.

used as a resistance standard, from a pure resistance equal to its direct current value.

To increase the power carrying capacity of the exponential line, one was built with larger wire size at the lower impedance end. This increased the breakdown voltage by increasing the spacing and conductor diameter and at the same time increased the current carrying capacity by decreasing the resistance and increasing the heat dissipat-

ing capacity of the conductors. This was a 600 : 300 ohm line constructed of 20 meters No. 6 wire, 10 meters 1/4″ tubing and 20 meters 3/8″ tubing. Here again the correct termination was determined by measurements on a 300 ohm uniform line of 3/8″ tubing. The total length of terminating loop that gave the best termination was 6½″ in this case compared with 10½″ for the 300 ohm line of No. 6 wire. Since no attempt was made to reduce the variations in input impedance to less than ± 1 per cent these lengths may be as much as an inch off.

These measurements indicated that the exponential line would perform satisfactorily as an impedance transformer if it could be constructed to have the desired mechanical features without impairing its electrical properties. The greatest difficulty appeared to reside in the

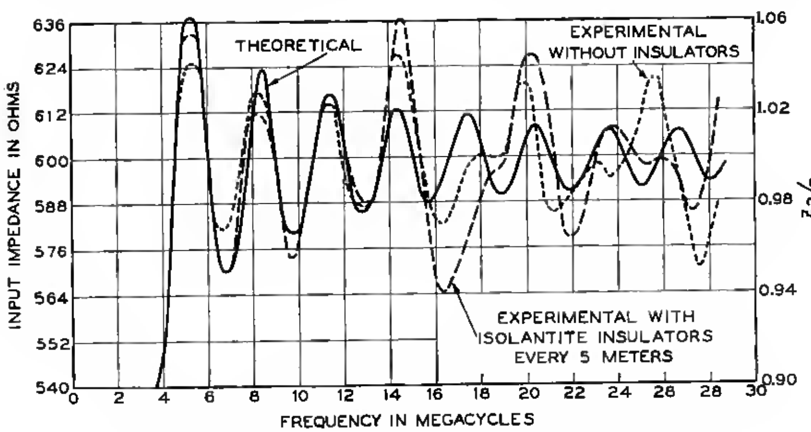

Fig. 8—Input impedance characteristics of 50-meter 600 : 300 ohm line of 3/8″, 1/4″ and No. 6 conductors.

insulators. Special isolantite insulators were designed that would be satisfactory commercially and still keep the additional capacity to a reasonable value. Figure 7 shows the construction of the line at the supporting poles where the conductor size changes.

The results of measurements on this line are shown in Fig. 8. The solid curve was calculated from the equations developed earlier. The two broken curves are the results of measurements on the line, one without insulators and one with insulators. While the insulators affect the line somewhat they do not increase the deviation from the ideal appreciably. [The improvement in the agreement between experiment and theory in this set of curves over that in Fig. 4 is presumably due to the fact that the comparison resistance for Fig. 8 consisted of 3-IRC

resistances instead of the pencil lead. With the fixed IRC resistance it was, of course, impossible to adjust the standard to exactly the same value as the unknown. In this case the small difference was determined by using the slope of the rectifier voltmeter calibration.] This line has a maximum deviation from the desired input impedance of ± 6 percent for all frequencies above 4.2 mc. (Measurements were made up to 28 mc.) The phase angle of the input impedance was found to be zero

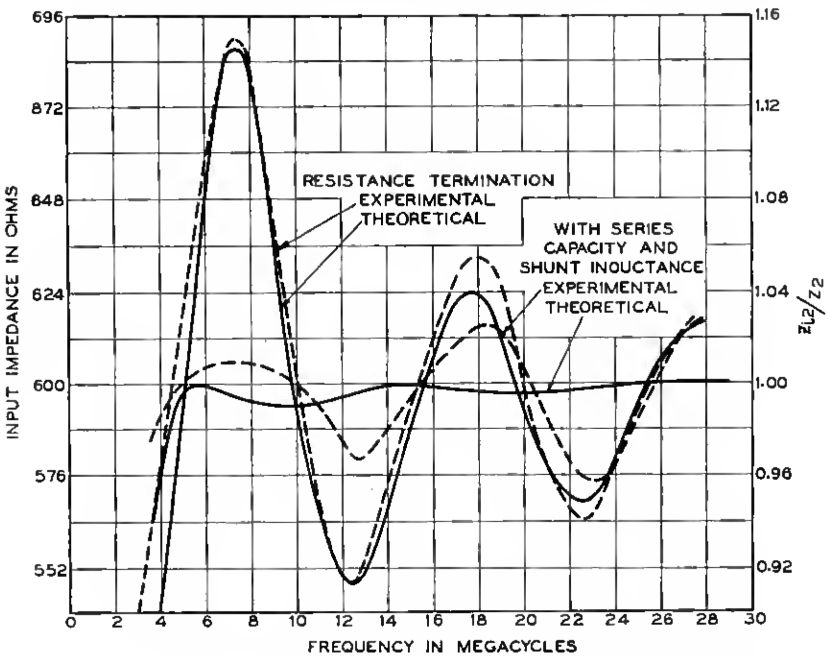

Fig. 9—Input impedance characteristics of 15-meter 600 : 300 ohm line of No. 6 conductors

within the accuracy of measurement. From theory the phase angle would be expected to vary between $-\ 0°$ and $+\ 3°$.

The curves of Fig. 9 refer to a 600 : 300 ohm line of No. 6 wire 15 meters long. With resistance termination this line has rather large variations in the input impedance but with the addition of the proper reactances the input impedance is flatter than the longer line with resistance termination. At the lower frequencies where the variations in the input impedance were large without the reactive networks, their addition gives approximately the expected improvement. At the higher frequencies the inductance was approximately anti-resonated

by its distributed capacity and the input impedance approaches that for the resistance termination.

Conclusion

Theory indicates that the exponential line may be used as an impedance transformer over a wide frequency range. The results of experiment show that the desired characteristic can be realized in practice. Among the applications of the exponential line may be mentioned its use in transforming the impedance level back to its original value after the paralleling of two transmission lines feeding two antennas. It could be used to transform the input impedance of a rhombic antenna down to the usual 600-ohm level of open wire transmission lines. If twin coaxial lines are used inside the transmitter building to eliminate undesired feedback, coupling, etc., the exponential line could be used to transform from the highest practical impedance level of such lines to a practical level of the more economical open wire lines for use outside the building.

Appendix

The exponential line is a non-uniform line so that the terms "characteristic impedance" and "surge impedance" of an exponential line are not synonymous. The terms "surge impedance" [1] and "nominal characteristic impedance" [2] may be used synonymously for the characteristic impedance of the uniform line that has the same distributed constants as the non-uniform line at the point in question. Expressed as functions of the distributed "constants" of the line they are the square root of the ratio of the distributed series impedance to the distributed shunt admittance at the point along the line in question. It will be expedient to refer to the nominal characteristic impedance as the impedance level at the point in question. Schelkunoff [3] has defined the characteristic impedances as the ratio of voltage to current at the point in question for each of the two traveling waves of which

[1] The term "surge impedance" is defined by A. E. Kennelly on page 73 of "The Applications of Hyperbolic Functions to Electrical Engineering Problems" (McGraw-Hill 1916) as follows: "The surge impedance of the line is not only the natural impedance which it offers everywhere to surges of the frequency considered, but it is also the initial impedance of the line at the sending end." Hence the "surge impedance" should be independent of the configuration of the line except at the point in question and in particular it should be equal to that for a uniform line constructed so as to have the same dimensions everywhere as the non-uniform line has at the point in question.

[2] The word nominal as used here has the same meaning as in "nominal iterative impedance" as used by K. S. Johnson in "transmission circuits for telephone communication" (Van Nostrand 1925).

[3] S. A. Schelkunoff, "The Impedance Concept and its Application to Problems of Reflection, Refraction, Shielding and Power Absorption," *Bell System Technical Journal*, *17*, 17–48, January, 1938.

the steady state condition is composed. At each point an exponential line has two characteristic impedances which are different and depend upon the frequency as well as the position along the line.

Because of the change of impedance level, the propagation constants for the voltage and current differ, so that it is convenient to consider the transfer constant [4] which may be defined as half the sum of the voltage and current propagation constants.

[4] Compare with the definition of "image transfer constant" as given by K. S. Johnson in "Transmission Circuits for Telephone Communication."